One or More Xs to One Y

3

Defining Functional Relationships

WARM UP

Evaluate each expression given the set of values {1, 6, 12, 25}.

1. $5x$
2. $\frac{1}{2}x + 1$
3. $x - 8$

LEARNING GOALS

- Describe a functional relationship in terms of a rule which assigns to each input exactly one output.
- Determine whether a relation (represented as a mapping, set of ordered pairs, table, sequence, graph, equation, or context) is a function.

KEY TERMS

- mapping
- set
- relation
- input
- output
- function
- domain
- range
- scatter plot
- vertical line test

Throughout middle school, you have investigated different types of relationships between variable quantities: additive, multiplicative, proportional, and non-proportional. What are functional relationships?

Getting Started

What's My Rule?

Rules can be used to generate sequences of numbers. They can also be used to generate (x, y) ordered pairs.

1. Write an equation to describe the relationship between each independent variable x and the dependent variable y. Explain your reasoning.

a.

x	y
−6	−12
−3	0
0	12
3	24

$y = 4x + 12$

b.

x	y
1	−2
5	−10
−1	2
−10	20

$y = -2x$

c.

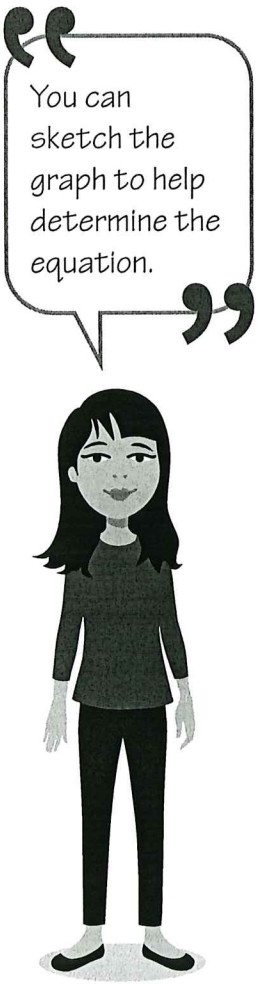
x	y
−10	9
−2	1
0	−1
5	4

$y = |x| - 1$

d.

x	y
0	2
4	4
5	4.5
20	12

$y = \frac{1}{2}x + 2$



2. Create your own table and have a partner determine the equation you used to build it.

ACTIVITY 3.1

Functions as Mappings from One Set to Another

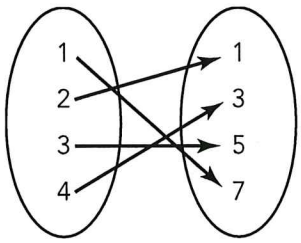
As you learned previously, ordered pairs consist of an *x*-coordinate and a *y*-coordinate. You also learned that a series of ordered pairs on a coordinate plane can represent a pattern. You can also use a *mapping* to show ordered pairs. A **mapping** represents two sets of objects or items. Arrows connect the items to represent a relationship between them.

When you write the ordered pairs for a mapping, you are writing a set of ordered pairs. A **set** is a collection of numbers, geometric figures, letters, or other objects that have some characteristic in common.

Use braces, { }, to denote a set.

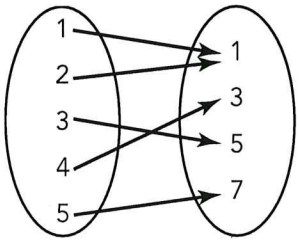
1. Write the set of ordered pairs that represent a relationship in each mapping.

a.



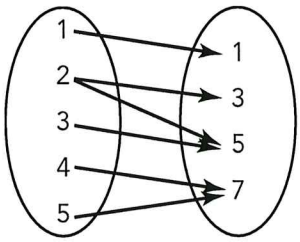
(1, 7)
(2, 1)
(3, 5)
(4, 3)

b.



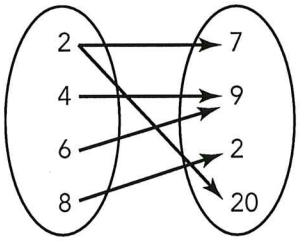
(1, 1)
(2, 1)
(3, 5)
(4, 3)
(5, 7)

c.



(1, 1)
(2, 3)
(2, 5)
(3, 5)
(4, 7)
(5, 7)

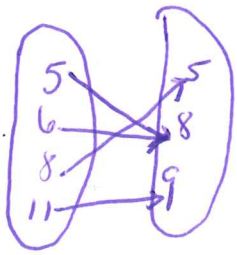
d.



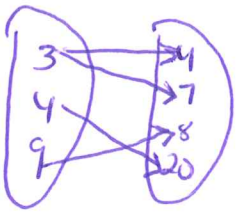
(2, 7)
(2, 20)
(4, 9)
(6, 9)
(8, 2)

2. Create a mapping from the set of ordered pairs.

a. {(5, 8), (11, 9), (6, 8), (8, 5)}



b. {(3, 4), (9, 8), (3, 7), (4, 20)}



3. Write the set of ordered pairs to represent each table.

a.

Input	Output
–10	–20
–5	–10
0	0
5	10
10	20

(-10, -20)
(-5, -10)
(0, 0)
(5, 10)
(10, 20)

b.

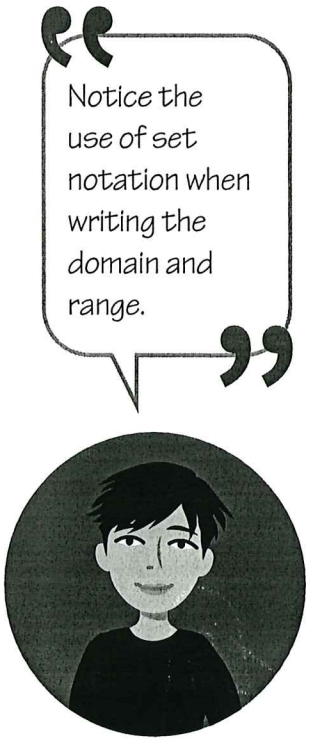
x	y
20	–10
10	–5
0	0
10	5
20	10

(20, -10)
(10, -5)
(0, 0)
(10, 5)
(20, 10)

x	y
Input	Output
Domain	Range
Independent	Dependent

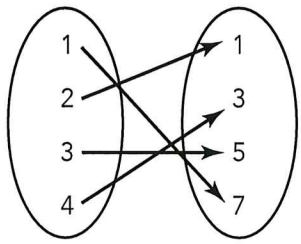
The mappings and ordered pairs shown in Questions 1 through 3 form *relations*. A **relation** is any set of ordered pairs or the mapping between a set of *inputs* and a set of *outputs*. The first coordinate of an ordered pair in a relation is the **input**, and the second coordinate is the **output**. A **function** maps each input to one and only one output. In other words, a function has no input with more than one output. The **domain** of a function is the set of all inputs of the function. The **range** of a function is the set of all outputs of the function.

Function – no x-value repeats

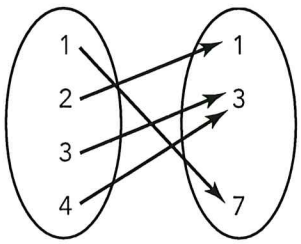


WORKED EXAMPLE

In each mapping shown, the domain is {1, 2, 3, 4}.



The range is {1, 3, 5, 7}.



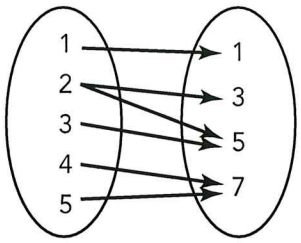
The range is {1, 3, 7}.

Each mapping represents a function because no input, or domain value, is mapped to more than one output, or range value.

NOTES

WORKED EXAMPLE

In the mapping shown, the domain is {1, 2, 3, 4, 5} and the range is {1, 3, 5, 7}.



This mapping does not represent a function.

4. State why the relation in the worked example shown is not a function.

The domain (x) of 2 maps to 3 & 5 in the range (y).

5. State the domain and range for each relation in Questions 2 and 3. Then, determine which relations represent functions. If the relation is not a function, explain why not.

Ques 2

(a) Domain {5, 6, 8, 11}
Range {5, 8, 9}
Function

(b) Domain {3, 4, 9}
Range {4, 7, 8, 20}
Not a function

Ques 3

(a) Domain {-10, -5, 0, 5, 10}
Range {-20, -10, 0, 10, 20}
Function

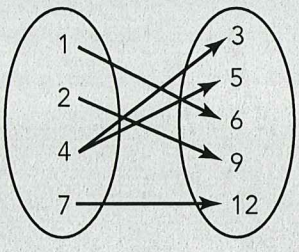
(3) Domain {0, 10, 20}
Range {-10, -5, 0, 5, 10}
Not a function

Think about the mappings as ordered pairs.

6. **Review and analyze Emil's work. Explain why Emil's mapping is not an example of a function.**

Emil

My mapping represents a function.



The input of 4 has two outputs. Therefore, it's not a function.

7. **Determine if each sequence represents a function. Explain why or why not. If it is a function, identify its domain and range. Create a mapping to verify your answer.**

* The terms are the outputs (y-values). The term # is the input (x-value).

a. **2, 4, 6, 8, 10, ...**

(1, 2)
(2, 4)
(3, 6)
(4, 8)
(5, 10)

Function

Remember that a sequence has a term number and a term value.

b. **1, 0, 1, 0, 1, ...**

(1, 1)(2, 0)(3, 1)(4, 0)(5, 1) - Function

c. **0, 5, 10, 15, 20, ...**

(1, 0)(2, 5)(3, 10)(4, 15)(5, 20) - Function.

ACTIVITY 3.2

Functions as Mapping Inputs to Outputs

You have determined if sets of ordered pairs represent functions. In this activity you will examine different situations and determine whether they represent functional relationships.

Read each context and decide whether it fits the definition of a function. Explain your reasoning.

1. ***Input:*** **Sue writes a thank-you note to her best friend.**
   Output: **Her best friend receives the thank-you note in the mail.**

   yes - her note goes to one place.

2. ***Input:*** **A football game is being telecast.**
   Output: **It appears on televisions in millions of homes.**

   No - the same game is sent to millions of different homes.

3. ***Input:*** **There are four puppies in a litter.**
   Output: **One puppy was adopted by the Smiths, another by the Jacksons, and the remaining two by the Fullers.**

   yes, each puppy has one and only one home.

4. ***Input:*** **The basketball team has numbered uniforms.**
   Output: **Each player wears a uniform with her assigned number.**

   Yes, each player wears one uniform with one number.

5. ***Input:*** **Beverly Hills, California, has the zip code 90210.**
   Output: **There are 34,675 people living in Beverly Hills.**

   No, One zip code is mapped to millions of people.

6. ***Input:*** **A sneak preview of a new movie is being shown in a local theater.**
   Output: **65 people are in the audience.**

   No one movie is mapped to 65 people.

7. *Input:* Tara works at a fast food restaurant on weekdays and a card store on weekends.
   Output: Tara's job on any one day.

yes - on any given day, she only works at one job.

8. *Input:* Janelle sends a text message to everyone in her contact list on her cell phone.
   Output: There are 41 friends and family on Janelle's contact list.

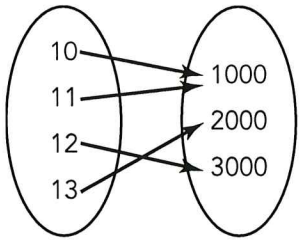
No - one text message goes to 41 people.

ACTIVITY 3.3

Determining Whether a Relation Is a Function

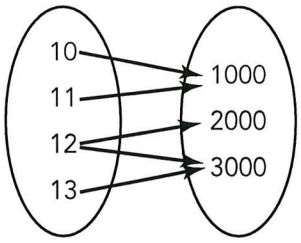
Analyze the relations in each pair. Determine which relations are functions and which are not functions. Explain how you know.

1. **Mapping A**



yes - each input has a unique output

Mapping B



No - 12 has two different outputs.

2. Table A

Input	Output
–2	4
–1	1
0	0
1	1
2	4

yes - no x repeats

Table B

x	y
2	–4
1	–1
0	0
1	1
2	4

No - 2 & 1 each have two different outputs.

3. Sequence A
7, 10, 13, 16, 19, ...

yes

Sequence B
10, 30, 10, 30, 10, ...

yes

★ All sequences are functions. No term #'s ever repeat.

4. Set A
{(2, 3), (2, 4), (2, 5), (2, 6), (2, 7)}

No - the #2 is mapped to five different outputs.

Set B
{(2, 1), (3, 1), (4, 1), (5, 1), (6, 1)}

yes - The y-values can repeat, The x-values cannot.

5. Scenario A
Input:
The morning announcements are read over the school intercom system during homeroom period.

Output:
All students report to homeroom at the start of the school day to listen to the announcements.

No - the same announcement goes out to several homerooms.

Scenario B
Input:
Each student goes through the cafeteria line.

Output:
Each student selects a lunch option from the menu.

yes - each student sets one lunch.

ACTIVITY 3.4

Functions as Graphs

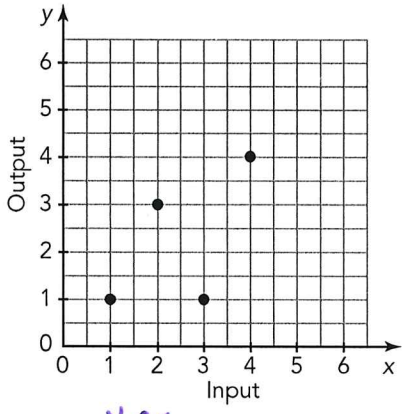
A relation can be represented as a graph.

A **scatter plot** is a graph of a collection of ordered pairs that allows an exploration of the relationship between the points.

1. Determine if each scatter plot represents a function. Explain your reasoning.

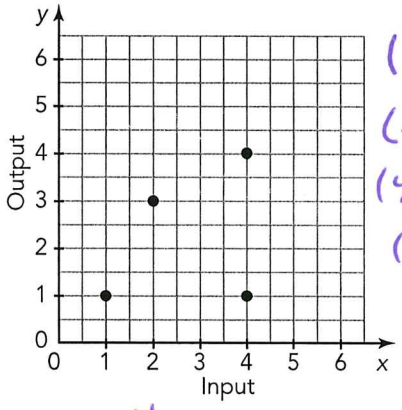
(1,1)
(2,3)
(3,1)
(4,4)

a.



yes

b.



(1,1)
(2,3)
(4,1)
(4,4)

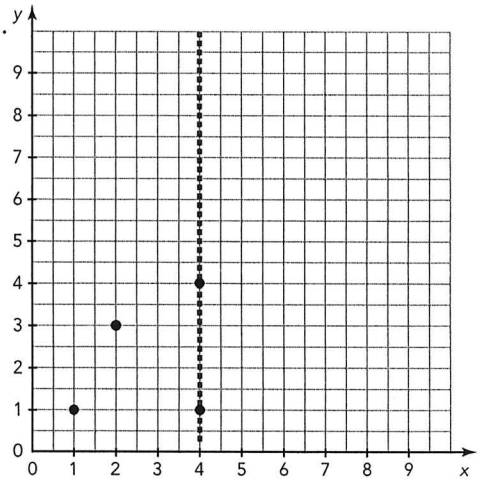
No

The **vertical line test** is a visual method used to determine whether a relation represented as a graph is a function. To apply the vertical line test, consider all of the vertical lines that could be drawn on the graph of a relation. If any of the vertical lines intersect the graph of the relation at more than one point, then the relation is not a function.

If a vertical line goes through more than one data point – it's not a function.

WORKED EXAMPLE

Consider the scatter plot shown.



In this scatter plot, the relation is not a function. The input value 4 can be mapped to two different outputs, 1 and 4. Those two outputs are shown as intersections to the vertical line drawn at $x = 4$.

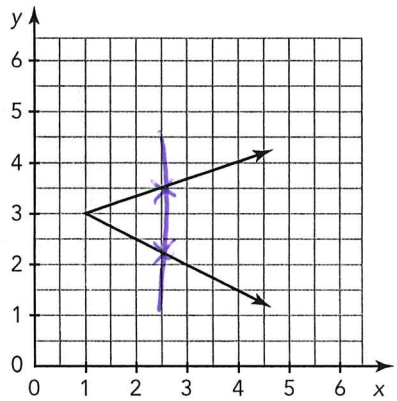
NOTES

2. Use the definition of function to explain why the vertical line test works.

If a vertical line goes through 2 points, then one x-value is mapped to two different y-values.

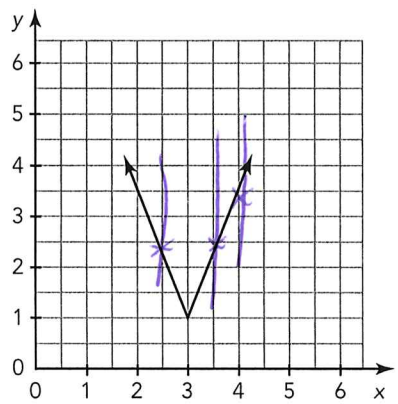
3. Use the vertical line test to determine if each graph represents a function. Explain your reasoning.

a.



No

b.



yes

4. Use the 12 cards that you sorted in the previous lesson. Sort the graphs into two groups: functions and non-functions. Use the letter of each graph to record your findings.

Functions	Non-functions
A, B, C, E, G, H, K, L	D, F, I, J

NOTES

ACTIVITY 3.5

Functions as Equations

So far, you have determined whether a mapping, context, or a graph represents a function. You can also determine whether an equation is a function.

WORKED EXAMPLE

The given equation can be used to convert yards to feet. Let x represent the number of yards, and let y represent the number of feet.

$$y = 3x$$

To test whether this equation is a function, first, substitute values for x into the equation, and then determine if any x-value can be mapped to more than one y-value. If each x-value has exactly one y-value, then it is a function. Otherwise, it is not a function.

x	$y = 3x$
1	3
3	9
4	12
8	24

In this case, every x-value can be mapped to only one y-value. Each x-value is multiplied by 3. Some examples of ordered pairs are (2, 6), (10, 30), and (5, 15). Therefore, this equation is a function.

It is not possible to test every possible input value in order to determine whether or not the equation represents a function. You can graph any equation to see the pattern and use the vertical line test to determine if it represents a function.

1. **Determine whether each equation is a function. List three ordered pairs that are solutions to each. Explain your reasoning.**

If you do not recognize the graph of the equation, use a graphing calculator to see the pattern.

a. $y = 5x + 3$ Yes

x	y
-1	-2
0	3
1	8

$5(-1)+3 = -5+3 = -2$
$5(0)+3 = 0+3 = 3$
$5(1)+3 = 5+3 = 8$

b. $y = x^2$ Yes

c. $y = |x|$ Yes

d. $x^2 + y^2 = 1$ No

e. $y = 4$ Yes

f. $x = 2$ No

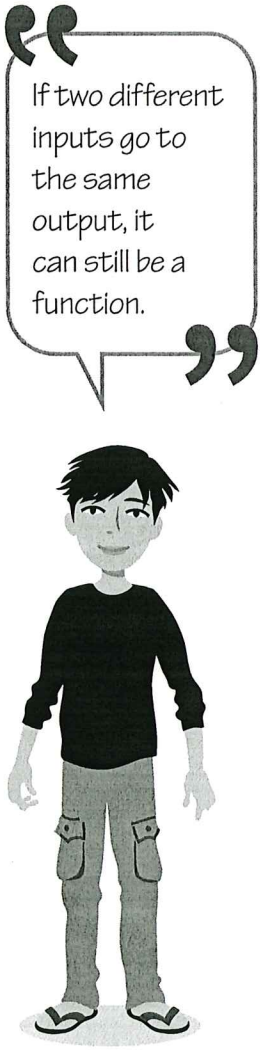
2. **Explain what is wrong with Taylor's reasoning.**

Taylor

The equation $y^2 = x$ represents a function.

x	y
4	2
9	3
25	5
9	-3
25	-5
4	-2

He forgot to try negatives.

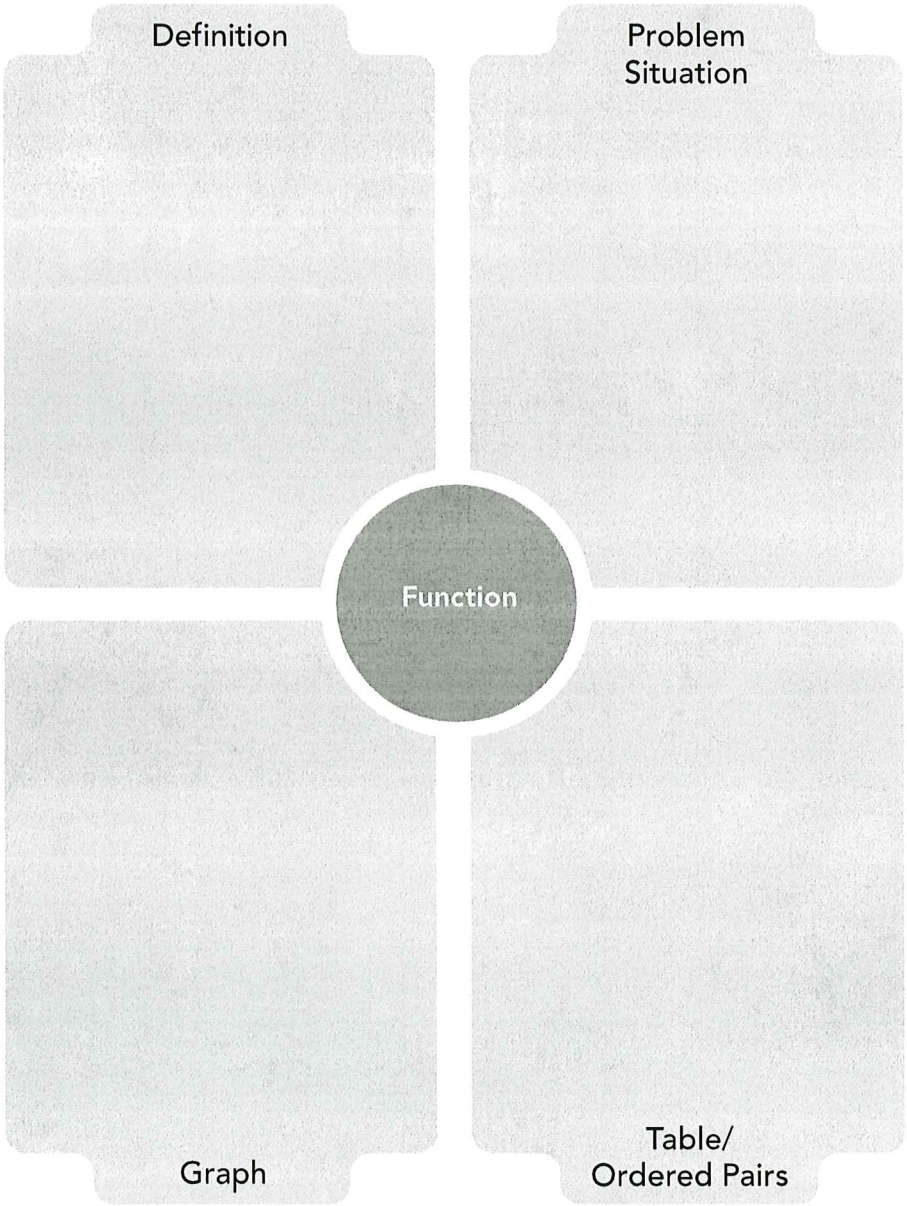


NOTES

TALK the TALK

Function Organizer

1. Complete the graphic organizer for the concept of function. Write a definition for *function* in your own words. Then, create a problem situation that can be represented using a function. Finally, create a table of ordered pairs and sketch a graph to represent the function.



Assignment

Write

Write the term from the box that best completes each sentence.

scatter plot	output	relation	input	vertical line test
mapping	set	domain	range	function

1. A(n) _____ is any set of ordered pairs or the mapping between a set of inputs and a set of outputs.
2. The first coordinate of an ordered pair in a relation is the _____.
3. The second coordinate of an ordered pair is the _____.
4. A(n) _____ maps each input to one and only one output.
5. A(n) _____ is a graph of a collection of ordered pairs.
6. The _____ is a visual method of determining whether a relation represented as a graph is a function by visualizing whether any vertical lines would intersect the graph of the relation at more than one point.
7. A(n) _____ shows objects in two sets connected together to represent a relationship between the two sets.
8. A(n) _____ is a collection of numbers, geometric figures, letters, or other objects that have some characteristic in common.
9. The _____ of a function is the set of all inputs of the function.
10. The _____ of a function is the set of all outputs of the function.

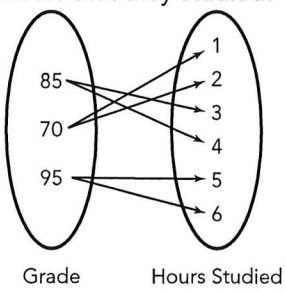
Remember

A relation is any set of ordered pairs or the mapping between a set of inputs and a set of outputs.

A relation is a function when each input value maps to one and only one output value.

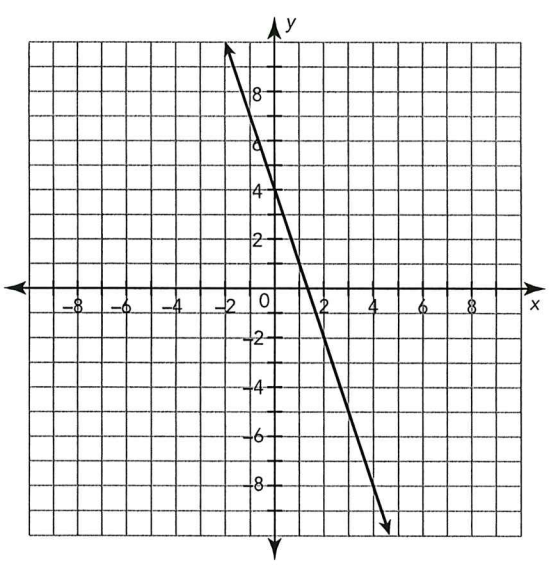
Practice

1. A history teacher asks six of her students the number of hours that they studied for a recent test. The diagram shown maps the grades that they received on the test to the number of hours that they studied.
   a. Is the relation a function? If the relation is not a function, explain why not.
   b. Write the set of ordered pairs to represent the mapping.
   c. What does the first value in each ordered pair in part (b) represent? What does the second value in each ordered pair represent?
   d. Create a scatter plot. Does the graph agree with your conclusion from part (a)? Explain your reasoning.

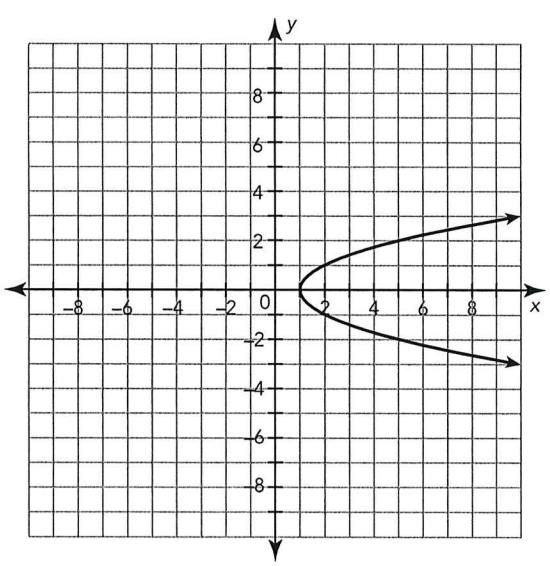


2. The science teacher created the set of ordered pairs {(100, 6), (90, 5), (80, 3), (70, 1), (90, 4), (80, 2)} to represent six students' grades on the midterm to the number of hours that they had studied. Create a mapping from this set of ordered pairs.
   a. Is the relation a function? If the relation is not a function, explain why not.
   b. List all the inputs of the relation.
   c. List all the outputs of the relation.
   d. Instead of mapping grades to hours studied, the teacher decides to create a new diagram. This diagram maps hours studied to grades. Show the mapping that would result.
   e. Write the set of ordered pairs to represent the mapping in part (d).
   f. Is the relation in part (d) a function? If the relation is not a function, explain why not.
   g. Create a scatter plot. Does the graph agree with your conclusion from part (f)? Explain your reasoning.
3. At the end of the year, a principal decides to create the given mapping.
   Input: the 82 total students in the history class
   Ouput: the final grades they received for the class
   Does this mapping fit the definition of a function? Explain your reasoning.
4. Use the vertical line test to determine if each graph represents a function. Explain your reasoning.

a.



b.



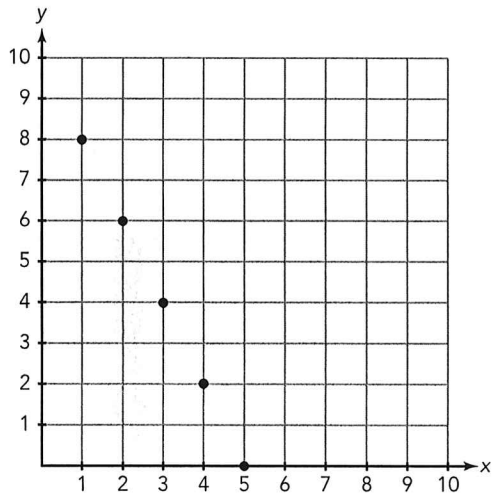
Stretch

Describe how you can tell from an equation whether a function is increasing, decreasing, or constant.

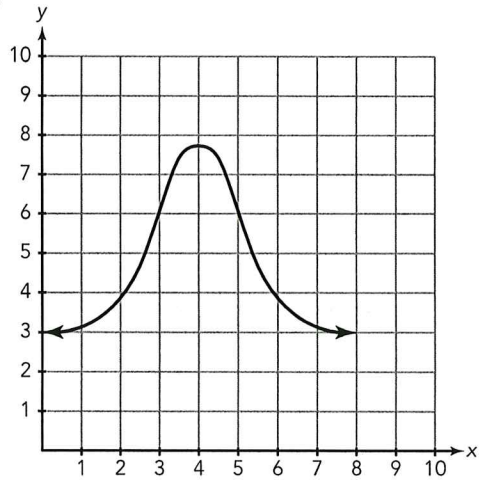
Review

Tell whether each graph is discrete or continuous. Also, tell whether each graph is increasing, decreasing, both, or neither.

1.



2.



Determine the slope and y-intercept of the linear relationship described by each equation.

3. $y = \frac{x}{2} + 5$

4. $y = \frac{x}{4}$

Calculate the slope of the line represented by each table.

5.

x	y
2	–1
3	1.5
4	4
5	6.5

6.

x	y
2	8
4	2
6	–4
9	–13